A simple energy budget algorithm for the snowmelt runoff model

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Abstract. The snowmelt runoff model (SRM) uses a degree-day approach for melting snow in a basin. A simple radiation component was combined with the degree-day approach (restricted degree-day method) in an effort to improve estimates of snowmelt and reduce the need to adjust the melt factor over the ablation season. A daily energy balance model was formulated that requires not only the input of radiation but also measurements of daily wind speed, air temperature, and relative humidity. The three approaches for computing snowmelt, namely, the degree-day, restricted degree-day, and daily energy balance model were tested at the local scale by comparing melt rates with lysimeter outflow measurements. Because radiation measurements are not often available, a simple model for simulating shortwave and longwave components of the radiation balance that requires minimal information (i.e., daily cloud cover estimates, air temperature, and relative humidity) was developed. It was found that clouds and their effects on daily insolation at the surface can produce significant differences between measured and model estimates. In the comparisons of snowmelt estimates with the lysimeter outflow, the restricted degree-day method yielded melt rates that were in better agreement with the observed outflow than the degree-day method and were practically the same as estimates given by the energy balance model. A sensitivity analysis of runoff generated with SRM using as input the local snowmelt computations given by the three models and measured outflow from the lysimeter was performed for a basin. A comparison of the synthetic hydrographs for the basin suggests that a radiation-based snowmelt factor may improve runoff predictions at the basin scale.

Introduction

The occurrence of precipitation in the solid form (snow) as opposed to the liquid form (rain) typically causes a change in drainage basin response to the input of water because snow is stored in a basin for an extended period of time before it enters into the runoff process. At the end of the winter accumulation period the seasonal snow cover melts in several weeks to several months depending on the amount of snow and location. When precipitation is snow rather than rain, evaporation losses during the runoff period are usually reduced.

Snowmelt runoff is especially important in the mountainous western United States, where water supplies are scarce and water demand is high. Most mountainous basins where snow occurs generally have at least 50% of their water supply produced by snowmelt. In certain localities, 95% of the water supply is derived from snowmelt in the spring and early summer [Shafer et al., 1982].

Snow on a drainage basin can cause significant changes in the energy exchange. Because of its high albedo, snow reflects a much higher percentage of incoming solar shortwave radiation than snow-free surfaces (of the order of 80% versus 15%). This causes a large difference in energy absorption and subsequent thermal heating. Additionally, the low thermal conductivity of snow greatly reduces heat exchange between the ground and the atmosphere so that snow serves as an insulating blanket for the underlying surface.

Energy balance models can account for many of the physical processes that affect snowmelt, but the input data to run the models are rarely available. As a result, the use of average temperature above freezing as an index of energy available for snowmelt is common.

The degree-day method for snowmelt-runoff calculations has been in use in different ways for almost 60 years [e.g., Collins, 1934]. In general, the degree-day method involves computing the daily snowmelt depth by multiplying the number of degree-days by the degree-day factor (see equation (1)). Linsley and Franzini [1979] point out that the factor generally increases as the melting period progresses, as observed earlier by Anderson [1968, 1976].

An advantage of the method is that it is easy to use operationally because a limited amount of data are required, usually only daily maximum and minimum temperatures. The method has been shown many times to produce accurate runoff predictions for alpine and subalpine drainage basins [World Meteorological Organization, 1986]. Because temperature is one of the key climate variables to be affected by climate change, the degree-day approach is easily adaptable...
to evaluation of various climate change scenarios associated with a temperature change [e.g., Rango and van Katwijk, 1990a; Martinec and Rango, 1989].

The snowmelt runoff model (SRM) improves upon the traditional degree-day approach by using remotely sensed observations of the snow-covered area. The basin is divided into elevation zones, and the remote observations are used to specify the portion of each elevation zone where the degree-day approach should be applied to melt the existing snowpack. As the snowpack is depleted, the degree-day approach is applied to only those portions of the basin where snow remains. The areal distribution employed in SRM provides a more physically based application of the degree-day approach.

A major drawback with this approach is that although the degree-day factor is a bulk melt factor that can provide a reasonably good measure of the total energy flux, other important melting factors like solar radiation, albedo, topography, and the turbulent energy exchange processes are not specifically taken in to account. Therefore the degree-day approach will be unreliable under conditions when these other factors may largely dominate the melt process [Anderson, 1968]. There is also high temporal and spatial variability in melt rates associated with these factors, which are not reflected in the degree-day approach. Therefore any improvements that can be added to the physical basis of the degree-day method will likely improve its use in both snowmelt runoff forecasting and evaluation of the effects of climate change.

The objective of the present work is to develop a more physically based snowmelt factor for SRM by combining a simplified radiation balance method with the degree-day method. In doing so it may be possible to restrict variation of the degree-day factor during the melt season. The study provides a comparison of snowmelt measurements at a point (lysimeter) with three approaches for calculating snowmelt, namely, the normal degree-day method, a restricted degree-day and radiation balance approach, and a total energy balance approach. Further evaluation is carried out by comparing snowmelt hydrographs from a watershed using SRM with the different snowmelt calculation methods.

Theory and Model Formulations

Degree-Day Method

The degree-day method is a temperature index approach that equates the total daily decrease of the water equivalent of the snowpack to a temperature difference between the daily average air temperature and a base temperature (usually 0øC). The coefficient multiplying this temperature difference is the degree-day factor [e.g., Westerstrom, 1982]:

\[ M = a(T_a - T_b) \] (1)

where \( a \) is the degree-day factor (centimeters per degree Celsius per day), \( T_a \) is average daily temperature (degrees Celsius), \( T_b \) is the base temperature (degrees Celsius), and \( M \) is the snowmelt rate (centimeters per day). The temperature difference in (1) is represented by \( T_d \), the degree-day temperature. When \( T_a < T_b \), then \( T_d = 0 \), and no melt will occur. Equation (1) has been widely used for short-term prediction of snowmelt [Martinec, 1960; Anderson, 1968; Granger and Male, 1978; Kuusisto, 1980; Moussavi et al., 1989].

However, because (1) implicitly accounts for all terms of the energy budget that affect the mass balance of the snowpack, \( a \) has high spatial and temporal variability. Under most circumstances the range in the degree-day factor is between 0.35 and 0.60 cm °C\(^{-1}\) d\(^{-1}\). Attempts have been made to adjust the value of \( a \) using physical parameters which affect the melt process and that can be easily measured or estimated. These include snow density [Martinec, 1960; Kuusisto, 1980], average daily air temperature range [Moussavi et al., 1989], and mean wind speed [Martinec, 1960].

Restricted Degree-Day Radiation Balance Approach

In the current study, net radiation was chosen because it appears to explain most of the variation in snowmelt [Zuzel and Cox, 1975; Granger and Male, 1978; Marks and Dozier, 1992]. The combination of the surface radiation budget and a temperature index has been proposed by several investigators [Martinec and de Quervain, 1975; Ambach, 1988; Martinec, 1989] and appears to satisfy the requirement for an operational yet more physically based snowmelt model:

\[ M = a_r T_d + m_Q R_n \] (2)

where \( a_r \) is a restricted degree-day factor, \( m_Q \) is the conversion factor for energy flux density to snowmelt depth (cm d\(^{-1}\) (W m\(^{-2}\))\(^{-1}\)), and \( R_n \) is net radiation (watts per square meter). The value of \( m_Q \) is approximately 0.026, so each 1 W m\(^{-2}\) of daily average energy input results in a daily snowmelt depth of about 0.03 cm water equivalent.

Preliminary findings by Martinec [1989] showed values of the restricted degree-day factor ranging between 0.20 and 0.25 cm °C\(^{-1}\) and exhibiting significantly less variability throughout the ablation period than the original degree-day factor. Lower values of \( a_r \) were generally found on days with low wind speeds, reducing sensible heat transfer, or with low humidity, which increased latent heat loss due to evaporation. The results of Martinec [1989] indicate that the restricted degree-day factor relates more to the physical processes it supposedly parameterizes, namely turbulent energy exchanges.

Surface Energy Balance Method

A statistical analysis by Zuzel and Cox [1975] indicated that daily snowmelt estimates improved by incorporating net radiation, vapor pressure, and wind in model formulations rather than air temperature alone. In other words, improvement in snowmelt predictions for a much wider range of conditions would result from computing the turbulent fluxes of latent and sensible heat [Anderson, 1976]. Hence the equation for estimating the energy available for snowmelt becomes

\[ \Delta Q = R_n + H + LE \] (3)

and is converted to snowmelt depth by

\[ M = m_Q \Delta Q \] (4)

where \( \Delta Q \) is the energy available for melt, \( H \) the sensible heat flux, and \( LE \) the latent heat flux, all in watts per square meter. Equation (3) is valid when the snowpack is isothermal at 0øC and its liquid water content is at the irreducible
satisfaction point. This is assumed to be a reasonable approxi-
mation during the ablation season.

In order to minimize the data and computational require-
ments, the simplified Thornthwaite-Holzman bulk transfer
approach for parameterizing the turbulent transfer of heat,
momentum, and water vapor was adopted, together with
stability criteria based on the bulk Richardson number Ri
[Morris, 1989]:

\[
H = \rho c_p C_h k^2 [\ln (z/z_0)]^{-2} u(\theta_a - \theta_s) \quad (5a)
\]
\[
LE = \rho L C_e k^2 [\ln (z/z_0)]^{-2} u(q_a - q_s) \quad (5b)
\]

where

\[
C_h = (1 - 58 Ri)^{0.25} \text{ for } Ri < 0
\]
\[
C_h = (1 + 7 Ri)^{-0.1} \text{ for } Ri > 0
\]
\[
C_e = 0.5 \ C_h
\]

and \(Ri = g z (\theta_a - \theta_s) (\theta_s u^2)^{-1}\), \(\rho\) is the density of air, \(c_p\) is
the specific heat of air at constant pressure, \(k\) is von
Karman’s constant (0.4), \(z_0\) is the roughness length for
momentum and scalars, \(L\) is the latent heat of vaporization,
\(\theta\) is the potential temperature, \(q\) is the specific humidity, \(u\) is
the wind speed at elevation \(z\) above the surface, \(g\) is the
gravitational acceleration, and the subscripts \(a\) and \(s\) represen-
t values at \(z\) and at the surface.

The bulk transfer approach is based on some simplifying
assumptions which include using similar roughness lengths
for momentum, heat, and water vapor and approximations to
the stability corrections via \(C_h\) and \(C_e\). However, because
daily inputs were used, the simplifications leading to (5) were
considered reasonable. Moreover, similar schemes have been
used with success in energy budget models for the prediction of
snowmelt in different environments [Dozier and O'Neil, 1979;
Granger and Male, 1978; Williams, 1988].

Simulating the Radiation Balance at an
Unobstructed Point at the Surface

In order to use the restricted degree-day factor or the
approach outlined in (3) and (4), the daily net radiation
balance at the surface must be determined. When there are
no radiation measurements, this requires a modeling ap-
proach whereby the shortwave and longwave components of
\(R_n\) can be simulated with minimal ancillary meteorological
data. This is discussed briefly below.

Global and Net Solar Radiation

Under clear sky conditions at a point on an unobstructed
horizontal surface, the total shortwave radiative flux re-
ceived per unit area (\(K\), global radiation) is the sum of
the direct insolation and the diffuse sky radiation [e.g., Kondra-
tyev, 1973]:

\[
K_o = K_{dir} + K_{dif} \quad (6)
\]

where \(K_o\) is the global radiation for clear skies, \(K_{dir}\) is the
direct solar radiation under cloudless conditions, and \(K_{dif}\) is
the diffuse component for clear skies. Note that in (6) and in
what follows the subscript \(o\) will always indicate cloudless
conditions. The direct and diffuse components under cloud-
less conditions were determined with an operational
approach described by Munro and Young [1982]. Numerical
integration as a function of solar zenith angle \(\alpha_s\) was
performed to obtain daily totals for \(K_{dir}\) and \(K_{dif}\), which
has been shown to be reasonably accurate [Garnier and
Ohmura, 1968, 1970; Olyphant, 1986b].

The expression for the direct radiation reaching the sur-
face under clear skies is

\[
K_{dir} = E_0 (1 - A_0) [t_{ad} - A_w] t_{ad} t_{sd} \quad (7)
\]

where \(t_{ad}\) is the transmission after absorption by dust, \(t_{sd}\) is
the transmission after scattering by dust, \(t_R\) is the transmis-
sion after Rayleigh scattering, \(E_0\) is the extraterrestrial
radiation, \(A_w\) is the absorption by water vapor (absorptiv-
ity), and \(A_0\) is the absorptivity of ozone. For water vapor,
expressions from Wang [1976] for the absorptivity, using
Kasten [1966] for computing the relative path length, were
employed along with the model developed by Brutsaert
[1975] for estimating the zenith path water vapor content.
For ozone the parameterizations of Lacs and Hansen [1974]
for absorptivity and Rodgers [1967] for the relative path
length were used. The quantity of ozone was estimated from
a formula developed by Van Heuklon [1979]. Values of \(t_{sd}\),
\(t_{ad}\), and \(t_R\) were obtained from expressions referenced by
Munro and Young [1982]. For diffuse radiation the formula
from Munro and Young [1982] reads

\[
K_{dif} = E_0 [0.5 (1 - A_0) (1 - t_a) + 0.8 (1 - A_0) t_{sd} (1 - t_{ad})]
\]

with the values 0.5 and 0.8 representing the scattering ratios
for Rayleigh and Mie scattering adopted from Davies and
Idso [1979].

In order to have a truly operational model the important
effects of clouds on the daily radiation balance need to be
simply parameterized. The approach developed by Munro
and Young [1982] was adopted where the direct radiation
under cloudy skies was calculated with

\[
K_{dir} = K_{dir} (1 - m_c) \quad (9)
\]

where \(m_c\) is the fractional cloud cover.

Diffuse radiation under cloudy skies, \(K_{dif}\), was computed
by adding the contribution of diffuse radiation from the
fraction of the sky which is cloud-free and that passing
through the cloudy portion, \(K_{dif}\), to the contribution of
diffuse radiation by multiple reflection between the cloud
cover and surface, \(K_{dif}\). Thus

\[
K_{dif} = K_{dir} + K_{dif} \quad (10)
\]

where

\[
K_{dif} = K_{dif} (1 - m_c) + m_c K_{dif} (t_{mc} - \alpha_{cl}) \quad (11)
\]

\[
K_{dif} = (K_{dir} + K_{dif}) m_c \alpha_{cb} \alpha_{dif} (1 - \alpha_{cb} \alpha_{dif})^{-1} \quad (12)
\]

The symbol \(\alpha_{cl}\) is the cloud top albedo, \(\alpha_{cb}\) is the cloud base
albedo, \(\alpha_{dif}\) is the surface albedo for diffuse radiation, and
\(t_{mc}\) is the cloud transmission. Values of \(\alpha_{cb}\) (0.6) and \(t_{mc}\)
(0.82) were taken from Munro and Young [1982], and the
equation from Fritz [1954], modified by Munro and Young
[1982], was adopted for estimating \(\alpha_{cl}\). Values of \(\alpha_{dif}\) were
determined with a snow grain size algorithm discussed
below.
During the snowmelt season the fraction of incoming radiation reflected from the surface represented by the broadband albedo, $\alpha_{sfc}$, decreases between snowfalls from around 0.9 to about 0.5 as a result of grain growth, contamination, and shallow depth. Typical snow albedo decay functions for the ablation season have been derived [e.g., Anderson, 1968, 1976; Petzold, 1977]. However, to determine $\alpha_{sfc}$ requires estimates of the diffuse surface reflectivity for direct radiation, $\alpha_{dir}$, and diffuse radiation, $\alpha_{diff}$. For the present study an approach by Williams [1988] was adopted which uses a physically based broadband parameterization for the snow albedo. This scheme implicitly accounts for some of the distinct spectral properties of snow reflection via separate determination of the reflectivity for direct and diffuse radiation under all sky conditions:

$$\alpha_{dir} = \alpha_{dir} - (0.083 + 0.23E_{1/2}) \cos^{1/2} \alpha_s$$  \hspace{1cm} (13)

$$\alpha_{diff} = \alpha_{diff} - 0.21E_{1/2}$$  \hspace{1cm} (14)

where

$$E = (E_s^3 + 0.252D_j)^{1/3}$$  \hspace{1cm} (15)

where $E$ is the mean grain radius (millimeters), $\alpha_s$ is the solar zenith angle, $E_s$ is the mean grain radius before melting occurs, usually between 0.1 and 0.4 mm, $D_j$ is the number of days since last snowfall, $\alpha_{dir}$ is the diffuse surface reflectivity for direct radiation at sunrise or sunset (=0.965), and $\alpha_{diff}$ is the surface reflectivity of diffuse radiation of fresh, dry snow (=0.96). The above formulation implicitly assumes that the surface of a melting snowpack is wet during the entire day and that each new snow accumulation consists of dry uncontaminated snow with the mean grain radius of new snow. Clearly, both the fraction of a day that the snow surface is wet and the mean grain radius are related to the amount of radiation absorbed and other feedback mechanisms implicit in the energy budget of the snowpack [Dozier et al., 1989]. Fortunately, (13) and (14) are not very sensitive to changes in the empirical factor used in (15) for estimating the changes in snow grain radius due to melting.

The above parameterizations summarized in (6)-(15) yield an estimate of the daily net shortwave radiation balance $K_n$ on an unobstructed horizontal surface:

$$K_n = (1 - \alpha_{dir})K_{dir} + (1 - \alpha_{diff})K_{diff} = (1 - \alpha_{sfc})K$$  \hspace{1cm} (16)

**Longwave Radiation**

Longwave radiation in the terrestrial environment originates mainly from two sources: emission from the atmosphere and emission from the Earth's surface. The longwave emission from the surface is proportional to the fourth power of its absolute temperature according to Stefan-Boltzmann's relation [e.g., Liou, 1980] and can be expressed by the following formula:

$$L_{sfc} = \varepsilon \sigma T_s^4$$  \hspace{1cm} (17)

where $\varepsilon$ is the surface emissivity, $\sigma$ is Stefan-Boltzmann's constant (5.67 x 10^{-8} W m^{-2} K^{-4}), and $T_s$ is the absolute surface temperature (kelvin). For snow the emissivity is between 0.98 and 0.99 [Dozier and Warren, 1982; Kondratyev et al., 1982] and is not significantly affected by changes in snow cover properties resulting from grain growth or contamination during the snowmelt season [Dozier and Warren, 1982]. Furthermore, because it is assumed that snowmelt occurs when there is a net energy input after the snowpack becomes isothermal at 0°C, $T_s$ was taken as a constant (i.e., 273 K) throughout the melt season. This yielded a mean value for $L_{sfc}$ of the order of 300 W m^{-2}.

The downwelling longwave radiation from the atmosphere, $L_{sky}$, cannot be described analytically. Although radiation models with varying degrees of complexity for computing $L_{sky}$ have been developed and tested [e.g., Luther et al., 1988], they are still too complicated for use in the present model. Approaches involving the use of screen level air temperature or vapor pressure or both for estimating the atmospheric emission are much more applicable [e.g., Brun, 1932; Swinbank, 1963; Idso and Jackson, 1969; Brutsaert, 1975; Unsworth and Monteleone, 1975]. On the other hand, most of these expressions are empirical and hence contain "constants" which vary significantly with locality.

A noteworthy exception is the model of Brutsaert [1975], who took a more physically based approach. Brutsaert's analysis yielded the effective atmospheric emissivity under clear skies, $e_{skyo}$, as a function of both screen level vapor pressure $e_a$ (pascals) and air temperature $T_a$ (kelvin):

$$e_{skyo} = 0.642(e_a/T_a)^{1/7}$$  \hspace{1cm} (18)

Aase and Idso [1978] found that under freezing conditions, (18) generally underestimated the atmospheric emissivity. Satterlund [1979] derived an exponential formula containing both vapor pressure and air temperature that improved the agreement with measurements under freezing conditions.

However, Brutsaert's equation seems preferable because of its theoretical foundation and the fact that his derivation allows adjusting for a decreasing amount of atmospheric water vapor with increasing altitude [Uijlenhoet, 1992]. Furthermore, daily average air temperatures during the melt season generally are not significantly below 0°C; hence (18) would not yield significant errors. Thus the value of $L_{sky}$ is evaluated with the following formula:

$$L_{sky} = e_{skyo} \varepsilon T_a^4$$  \hspace{1cm} (19)

By combining (17) and (19) the net longwave radiation under clear skies, $L_{no}$, for an unobstructed horizontal surface can be written as follows:

$$L_{no} = \varepsilon (e_{skyo} T_a^4 - T_s^4)$$  \hspace{1cm} (20)

For cloudy conditions the atmospheric emission increases, mainly as a result of the increased water vapor content. Some investigators [e.g., Brutsaert, 1982; Unsworth and Monteleite, 1975] have presented correction factors for $L_s^4$; but this assumes $L_n$ is negative, which typically is not the case for snow-covered surfaces during the melt season. Thus an adjustment to the effective atmospheric emissivity would seem more appropriate for radiation modeling over snow.

For operational approaches in the near future it is likely that daily cloud cover observations will be the only information routinely available. Therefore in the present study the atmospheric emissivity for clear skies was corrected for the effect of cloudiness via a nonlinear function of the mean fractional cloud cover [Brutsaert, 1982]:

$$e_{sky} = e_{skyo}(1 + cm^2)$$  \hspace{1cm} (21)
where \( m_c \) is the fractional cloud cover and \( c \) is an empirical coefficient. Although the coefficient \( c \) in (21) is dependent on cloud type, Brutsaert [1982] suggested that \( c = 0.22 \) is a satisfactory mean value. This is supported by the experimental findings of Kimball et al. [1982]. Substitution of (21) into (20) yields a daily estimate of the net longwave flux for an unobstructed horizontal surface:

\[
L_n = \varepsilon \sigma (\varepsilon_{sky} (1 + c m_c^2) T_{a0}^4 - T_s^4)
\]

(22)

By combining (16) for the net daily shortwave flux \( K_n \) with (22) for the net longwave flux \( L_n \), the expression for computing the daily net radiation for a horizontal unobstructed surface is obtained:

\[
R_n = K_n + L_n = (1 - \alpha_{dir}) K_{dir} + (1 - \alpha_{dif}) K_{dif}
+ \varepsilon \sigma (\varepsilon_{sky} T_{a0}^4 - T_s^4)
\]

(23)

It should also be mentioned that satellite remote sensing for assessing cloud properties is showing some promise [Rossow, 1989] and may one day provide the necessary information for more physically based modeling. Moreover, estimation of surface insolation and the net shortwave radiation balance [e.g., Chou, 1989; Darnell et al., 1988; Dedieu et al., 1987] as well as net radiation [Pinker and Tarpley, 1988] from satellite remote sensing is another potential source of input data for model calibration and verification of catchment and regional scale computations (for a recent review see Sellers et al. [1990]). Nevertheless, in the present model formulation such approaches are not yet suitable from an operational standpoint.

### Modeling Radiation Over Complex Terrain

Difficulties in modeling the radiation balance in mountainous terrain are mainly associated with topographic effects which modify the incident radiation via obstruction, reflection, and emission by adjacent surfaces. These factors are more prevalent in alpine watersheds where most of the larger snow-covered areas have slopes of 10° to 30° and significantly influence the radiation balance [Olyphant, 1986a].

For the present study, no attempt has been made to include the effects of adjacent terrain and slope/aspect on the radiation balance at a point. Models for solar [e.g., Dozier, 1980; Olyphant, 1984] and thermal [e.g., Marks and Dozier, 1979; Olyphant, 1986a] radiation of such complexity have been developed and implemented with digital elevation data. However, at present these algorithms are difficult to include in an operational mode due to the amount and nature of input data (e.g., cloud cover) and the computational requirements.

On the other hand, studies improving the efficiency in spatial integration algorithms [Dozier and Frew, 1989] and extrapolation of input data [Running et al., 1987] will inevitably allow more detailed treatment of the radiation balance in complex terrain.

As a means of incorporating some of the major terrain effects on radiation, Uijlenhoet [1992] developed a submodel for computing the radiation budget for an infinitely long V-shaped valley. Thus only gross features of a watershed, such as basin orientation and average slope and elevation, are required. The radiation budget for a watershed using this idealized case will be tested in a future study.

### Evaluation of Snowmelt Algorithms

#### Experimental Data Set

The performances of the degree-day, restricted degree-day, and the bulk transfer/energy balance approaches for estimating daily snowmelt were assessed using outflow measurements from a snow lysimeter at the test site of the Swiss Federal Institute for Snow and Avalanche Research at Weissfluhjoch/Davos, Switzerland (46.8°N, 9.8°E). The snow lysimeter, which has a surface area of 5 m², is situated in a horizontal snow field at an altitude of 2540 m above mean sea level. The snow lysimeter is an advanced type featuring high side walls (60 cm) to prevent lateral inflows and improved subsurface drainage on the experimental plot compatible with the surrounding landscape. Its outflow is intercepted by a steel vessel and recorded continuously using a tipping bucket gauge. Due to resistance in the unsaturated and saturated snow layers and in the pipe leading from the vessel to the gauge, the transformation of a snowmelt depth resulting from a positive energy budget at the snow surface into an outflow hydrograph from the entire snowpack in the lysimeter exhibits a certain time lag and attenuation. However, it was found that, apart from daily fluctuations associated with daytime variations in snowmelt and nightly refreezing, the liquid water content of the snowpack did not increase any more after the day on which the lysimeter outflow started. These features of the snow lysimeter and concurrent checks of the hydrological balance at the experimental site indicate that the lysimeter measurements are representative of the snowmelt processes in the area [Martinec, 1987]. Consequently, one can assume on a daily basis that the change in snowmelt depth (water equivalent) of the site essentially equals lysimeter outflow [Martinec, 1989].

The snow lysimeter data set used for comparison with model simulations was collected during the 1985 ablation season by the Federal Institute for Snow and Avalanche Research. It consisted of daily averages of air pressure (\( p \)), air temperature (\( T_a \)), relative humidity (RH), wind speed (\( u \)), fractional cloud cover (\( m_c \)), sunshine duration (\( n \)), global radiation (\( K \)), precipitation occurring as snow (\( P_s \)), and precipitation occurring as rain (\( P_r \)). The entire 1985 snowmelt season lasted from May 9 (start of snowmelt) to July 15 (last day with lysimeter outflow). During the first week, however, no lysimeter outflow occurred because the entire snowmelt depth was used to increase the liquid water content of the snowpack gradually. Hence ripe conditions only occurred on the 58 days between May 16 (start of lysimeter outflow) and July 12, which therefore were taken into consideration.

The data pertaining to the mass (i.e., water) balance of the snow lysimeter were collected at a different location than the meteorological data. These data included \( p \), which was measured at an altitude of 2667 m, and \( T_a \), RH, \( u \), \( m_c \), and \( K \), which were evaluated at 2693 m above mean sea level (at the automatic meteorological station of the Swiss Meteorological Office). Hence the meteorological data are from 127 and 153 m above the snow field containing the lysimeter. Measurements of \( p \) and \( T_a \) were extrapolated downward to the snow field using standard atmosphere profiles. The value of RH was assumed to be constant over this altitude difference [Marks and Dozier, 1979], which allowed easy computation of the vapor pressure at the lysimeter and is basically
the same as assuming an exponential vertical vapor pressure profile [Brutsaert, 1975]. Although u will likely be overestimated when applied directly to the lysimeter because it was measured at a mountain summit, there was no alternative due to a lack of more appropriate data. No correction for elevation differences was made to mc, either, which is supported by the findings of Olyphant [1984]. Finally, K will be overestimated when applied directly to the lysimeter, because it was measured at a ridge top where the effects of obstruction by surrounding terrain are negligible. However, no correction was made to K because the effects of obstruction by neighboring surfaces to the amount of solar radiation received by the snow lysimeter could not be estimated due to a lack of appropriate topographic data.

The minima, maxima, averages, and standard deviations of some of the corrected input variables are listed in Table 1. From the table, note that the average vapor pressure of the air (ea = 605 Pa) during the ablation period is only slightly lower than the saturated vapor pressure over melting snow (ea = 611 Pa). The average is significantly higher than observations collected in other midaltitude alpine basins [Marks et al., 1992]. This may indicate that the humidity sensor was losing calibration and as a result overestimating humidity (D. Marks, personal communication, 1993). Also, there is significant variability in the magnitude of the mean fractional cloud cover (mcr) and, consequently, in the range in values of the daily average global radiation (K). The total lysimeter outflow (ΣQl) during the 58 nearly equilibrium days of the 1985 snowmelt season at the test site at Weissfluhjoch amounted to 98.6 cm, of which 13.6 cm can be attributed to discharge resulting from rainfall (ΣPr) and the remaining 85.0 cm, consequently, to actual snowmelt (ΣM).

| Table 1. Minima, Maxima, Averages, and Standard Deviations of Some of the Daily Average Input Variables Collected at the Weissfluhjoch Test Site During the 1985 Ablation Period |
|---|---|---|---|---|---|---|---|---|
| Minima | Maxima | Averages | Standard deviations |
| P, Pa | Ta, K | ea, Pa | u, m s⁻¹ | mc, m | K, W m⁻² | Pr, cm d⁻¹ | Ql, cm d⁻¹ |
| 739 | 757 | 749 | 4.0 | 79 | 693 | 10.0 | 1.0 |

See text for notation of symbols.

Evaluating Model Performance

To assess the simulation performance of the three previously described point snowmelt prediction methods, several different statistics were employed. The mean bias error (MBE) and the root mean square error (RMSE) [Willmott, 1982] were computed and expressed as a percentage of the measured mean. The coefficient of determination (r²) and the slope and the intercept (and the associated standard error) resulting from a linear regression analysis between the simulated and measured values were also evaluated.

Results

Instantaneous values of global radiation were generated according to the parameterization presented in (6)-(16). The instantaneous values were integrated numerically from sunrise to sunset using Simpson’s 1/3 rule with a temporal increment of 1 hour. The resulting daily averages of global radiation represented the measured values rather well on a seasonal average basis, as is indicated by a MBE of 2.35%. On a daily basis there were larger differences between measured and modeled values, as is indicated by a RMSE of 21%. A linear regression analysis of the simulated versus the measured values yielded 0.92 for the slope, 29 W m⁻² for the intercept, and 57 W m⁻² for the standard error. The value of r² = 0.56 indicated significant scatter between simulated and measured radiation (Figure 1). As can be seen from Figure 2, the daily simulated values seem to follow the overall trends in the measured values but generally overpredict high values and underpredict low values. The measured global radiation on days when the mean fractional cloud cover equalled 1 ranged from 57 to 365 W m⁻², varying by more than a factor of 6. This result plus the comparison in Figure 2 illustrate the problems associated with modeling the variability of global radiation due to cloud cover effects on a daily average basis without taking into account the diurnal variations or the cloud type.

The simulated daily average surface albedo decreased from about 0.85 for each new accumulation of fresh dry snow (with a mean grain radius of 0.2 mm) to 0.59 for saturated and contaminated snow at the end of the ablation period (Figure 3). The net shortwave radiation simulated by the model is quite sensitive to variation in the albedo decay during the snowmelt season. Furthermore, variability in the net longwave radiation can greatly influence the surface radiation budget. In the model the magnitude of the net longwave radiation is primarily affected by the type of formula for determining the effective atmospheric emissivity. A seasonal average net radiation of +19 W m⁻² was obtained, composed of +76 W m⁻² for the net shortwave radiation and −56 W m⁻² for the net longwave radiation. Thus the net shortwave radiation simulated by the model makes up nearly 58% of the radiation fluxes, leaving 42% contributed by the longwave balance. The minimum and maximum daily average net radiation were found to be −86 and +96 W m⁻², respectively.

The total simulated snowmelt occurring as a result of a positive radiation budget at the snow surface accounted for 54% of the lysimeter outflow during the entire ablation period. This is slightly smaller than 60% obtained by Martinec [1989] using measured radiation data for the same snowmelt season. Because the average energy flux density that is associated with cooling of precipitation received by the snowpack is less than 0.5 W m⁻² (with a total meltwater
equivalent of only 0.7 cm), it is reasonable to conclude that the remaining 46% of the cumulative lysimeter outflow is the result of net turbulent heat transfer between the snow surface and the atmospheric boundary layer. The value of $r^2$ between the measured lysimeter outflows and the simulated radiative melt was 0.78. The RMSE was found to be 60%, and the slope, intercept, and standard error of the performed linear regression analysis were 0.64, -0.13, and 0.43 cm d$^{-1}$, respectively. These statistics confirm the findings of several authors, who argue that although net radiation explains most of the variation in snowmelt, there is no simple proportionality between the two [Zuzel and Cox, 1975; Olyphant, 1984].

The seasonal averages of the simulated flux densities associated with the input of sensible heat and the loss of latent heat were 13 and $-0.8$ W m$^{-2}$, respectively. The latter is the result of a net loss of 1.3 cm of water equivalent from the lysimeter due to evaporation of meltwater, which is negligible in the mass balance of the snow lysimeter in this particular case. For the same site and ablation period, Martinec [1989] estimated 4.81 cm of evaporation of meltwater from the computation of hourly melt rates. This...
translates into an average loss of latent heat of \(-3.1 \text{ W m}^{-2}\) for the snowmelt season. According to the atmospheric stability criterion (see equation (5)) stable (near neutral) conditions prevailed throughout the snowmelt season (i.e., 43 of the total of 58 days). The correction factor to account for departures from neutral conditions never departed much from unity. The average restricted degree-day factor \((a_r)\) assessed to fit the simulated daily average turbulent transfer throughout the snowmelt season amounts to \(0.18 \text{ cm} \cdot \text{C}^{-1} \cdot \text{d}^{-1}\), which is close to the value of \(0.20 \text{ cm} \cdot \text{C}^{-1} \cdot \text{d}^{-1}\) that Martinec [1989] assessed for the same ablation period using measurements of global radiation instead of simulations.

In Figure 4 the seasonal variation of the energy budget terms in (3) are illustrated. Note the relatively small latent heat flux component and the fact that almost an equal number of positive and negative values exist. This behavior in \(LE\) is not consistent with estimates in a small alpine basin by Marks and Dozier [1992]. One cause may be instrumental. As discussed above, consistently high relative humidities measured by the sensor may indicate that it was losing its calibration. With (5b) such high humidities will significantly reduce the computed values of daily \(LE\). Another reason may be the use of daily data in (5a) and (5b) instead of meteorological observations available at a higher frequency.
Table 2. Summary Statistics Comparing Lysimeter Outflow Measurements With Simulations of the Daily Snowmelt According to the Original Degree-Day Method $a$, the Restricted Degree-Day Method $a_r$, and the Turbulent Transfer/Energy Balance Method $\Delta Q$

<table>
<thead>
<tr>
<th>Water Equivalent (Snowmelt)</th>
<th>MBE, %</th>
<th>RMSE, %</th>
<th>$r^2$</th>
<th>Slope cm d$^{-1}$</th>
<th>Intercept cm d$^{-1}$</th>
<th>Standard Error of the Estimated Water Equivalent, cm d$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>-1.8</td>
<td>65</td>
<td>0.54</td>
<td>0.82</td>
<td>0.23</td>
<td>0.94</td>
</tr>
<tr>
<td>$a_r$</td>
<td>-5</td>
<td>49</td>
<td>0.73</td>
<td>0.96</td>
<td>-0.02</td>
<td>0.72</td>
</tr>
<tr>
<td>$\Delta Q$</td>
<td>-7.4</td>
<td>50</td>
<td>0.74</td>
<td>0.99</td>
<td>-0.09</td>
<td>0.73</td>
</tr>
</tbody>
</table>

The input variables were collected at the test site at Weissfluhjoch during the 1985 ablation period.

The results in Table 2 suggest that relative to the degree-day method both the restricted degree-day and the bulk transfer/energy balance method significantly improve the agreement with measured snowmelt from the lysimeter. The relative reduction in RMSE is about 20%, and $r^2$ increases by nearly 40%. Overall, there is little difference between the restricted degree-day and the bulk transfer/energy balance method in simulating the snowmelt. This result is encouraging because more physically based models like the bulk transfer/energy balance approach outlined in (3)–(5) require atmospheric data (i.e., screen height, wind speed, and vapor pressure) which are not commonly available in alpine basins.

Apart from direct comparison between the simulated snowmelt depths and the measured lysimeter outflows during the 1985 snowmelt season at the Weissfluhjoch test site, a brief sensitivity analysis was carried out through the intercomparison of hydrographs generated for a complete watershed. The point snowmelt depths simulated according to the three methods and the outflows measured at the snow lysimeter were transformed into the respective runoffs that would occur from the nearby Dischma basin, provided that the inputs were representative for the whole basin. In other

![Figure 5. Daily meltwater equivalent measured by the lysimeter and simulated by the original degree-day method, the restricted degree-day method, and the bulk transfer/energy balance method through the 1985 snowmelt season at the Weissfluhjoch test site.](image-url)
Table 3. Summary Statistics Comparing the Simulation of Daily Discharge (Expressed as Water Equivalent) Using SRM for the Dischma Basin With Daily Snowmelt Given by the Lysimeter Versus the Original Degree-Day Method \(a\), the Restricted Degree-Day Method \(a_r\), and the Turbulent Transfer/Energy Balance Approach \(\Delta Q\) for Estimating Melt

<table>
<thead>
<tr>
<th>Water Equivalent (Flow)</th>
<th>MBE, %</th>
<th>RMSE, %</th>
<th>(r^2)</th>
<th>Slope</th>
<th>Intercept, cm d(^{-1})</th>
<th>Standard Error of the Estimated Water Equivalent, cm d(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>-1.6</td>
<td>35</td>
<td>0.82</td>
<td>0.72</td>
<td>0.19</td>
<td>0.19</td>
</tr>
<tr>
<td>(a_r)</td>
<td>4.0</td>
<td>25</td>
<td>0.91</td>
<td>0.97</td>
<td>0.05</td>
<td>0.18</td>
</tr>
<tr>
<td>(\Delta Q)</td>
<td>-3.8</td>
<td>25</td>
<td>0.91</td>
<td>0.94</td>
<td>0.02</td>
<td>0.17</td>
</tr>
</tbody>
</table>

words, the point melt rates given by the three models and measured by the lysimeter were assumed representative of the melt rate for the entire Dischma basin. These values were the estimates of the snowmelt rate \(M\) required in (24a).

Although the use of point inputs for representing area inputs obviously is a gross simplification [e.g., Blöschl et al., 1991], it will provide some insight into the sensitivity of the applied snowmelt-runoff transformation model to this data. The simplest form of the snowmelt runoff model (SRM), where the basin is not subdivided into different elevation zones, was used for this purpose [Martinec et al., 1983]:

\[
Q_{n+1} = c_n(M_nS_n + P_n)A(1 - k_n) + Q_nk_n (24a)
\]

where \(Q\) is the daily discharge (cubic meters per day), \(c\) is a runoff coefficient (=0.9), \(S\) is the ratio of snow-covered area to the basin area, \(P\) is the precipitation contributing to runoff (meters per day), \(A\) is the basin area (square meters), \(k\) is the recession coefficient, \(x\) is the recession factor (=0.85), \(y\) is the recession exponent (=0.086), and the subscript denotes the sequence of days during the discharge computation period. Note that the values used for the parameters in (24a) and (24b) are typical for the alpine Dischma basin, Switzerland, and are not applicable to other basins.

Representative parameter values for the Dischma basin in 1985 were obtained from a set of parameter values established using 10 years of SRM simulations on the Dischma basin [Martinec and Rango, 1986]. Although their publication shows that runoff coefficients for snowmelt and precipitation can differ markedly from each other, one value is applied for the current purpose because the contribution of rainfall will be small as compared to that of snowmelt. In SRM, new snowfall is just incorporated into the existing snowpack. The relative snow-covered area of the basin is assumed to decrease linearly from 1 to 0 during the snowmelt season, although analyses of data obtained from aircraft photography and satellite imagery show that areal snow cover depletion generally follows an S curve [Rango and van Katwijk, 1990b]. It can be seen from (24a) that during periods of true recession, \(k_{n+1} = Q_{n+1}/Q_n\). In SRM this recession coefficient is not assumed to be a constant as usual (leading to an exponential recession) but rather to be a function of the discharge on the day before, according to (24b).

The discharge results using the three methods for simulating snowmelt compared to the lysimeter outflow measurements are summarized in Table 3. Note that the discharge resulting from (24a) and (24b) is converted to equivalent water depths for convenience. It can be seen by comparing the results from Tables 2 and 3 that the snowmelt-runoff transformation given by (24a) and (24b) yields a relative reduction in the RMSE of all three methods by nearly 50% and increases \(r^2\) or the proportion of the variance of the measured water equivalents that is explained by the simulated values by nearly 25%. Furthermore, the standard errors of the estimated water equivalent values are reduced by an order of 4. The synthetic hydrographs produced by (24) are illustrated in Figure 6. Note that the largest differences occur during the peak runoff period, where melt rates are generally the highest (see Figure 5). This is a common result since the most dynamic period of the melt season is the most difficult to simulate. In evaluating the snowmelt-runoff process at the basin scale, sensitivity of simulated discharge to the three methods of computing snowmelt was reduced. Nevertheless, the findings summarized in Tables 2 and 3 indicate that including a radiation component in SRM to predict daily snowmelt generally improves model predictions. Similar conclusions have been reached in earlier studies [e.g., Tarboton et al., 1991] that combine radiation with temperature index approaches.

Conclusions

The result of this study showed that combining a radiation component with the temperature-based approach for estimating snowmelt generally improves results compared to using solely a temperature-based snowmelt factor and is in good agreement with a more complicated energy balance model. The comparison was carried out at the local scale with snowmelt estimates compared to lysimeter outflow measurements. Furthermore, runoff for a basin was generated with SRM using local snowmelt computations from the three models and using the outflow measurements from the lysimeter. These melt rates were assumed to be representative values for the whole basin. Comparison of the synthetic hydrographs associated with the three models versus the synthetic hydrograph using the lysimeter data suggests that a more physically based snowmelt factor may improve predictions at the basin scale.

The effects of clouds on the daily insolation at the surface...
are shown to produce significant differences between measurements and model estimates. While this scatter to some degree is due to the simplified approach used in the present formulation, errors associated with the ground observation of daily cloud cover also contributed to the disagreement [McGuffie and Henderson-Sellers, 1989]. Nevertheless, the radiation model in its present form cannot adjust daily insolation values without information on sky conditions. Computations of snowmelt by radiation are also sensitive to the estimated albedo values [e.g., Bloschl, 1991]. Therefore it may be more suitable to use a radiation-based approach for runoff simulations than for real time runoff forecasts because it is difficult to forecast cloudiness, albedo changes, etc. However, use of satellite remote sensing for estimating cloud cover, albedo, and insolation is another avenue which may one day have operational capabilities for providing real time data [e.g., Ewing and Pinker, 1988; Salby et al., 1991].

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